

Can One Study Heavy Meson Semileptonic Decays on Coarse Anisotropic Lattices ?

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Exploratory studies of heavy meson semileptonic decays on coarse lattices using Symanzik improved glue, NRQCD heavy and D234 light quarks are discussed. Comparisons are made between calculations on anisotropic and isotropic lattices. We find evidence that having an anisotropy helps in extracting better signals at higher momenta.

1. Introduction

Semileptonic decays of heavy mesons into light hadrons such as, $B \rightarrow \pi l \nu$ or $B \rightarrow \rho l \nu$, involve hadrons with sizeable momenta, once one moves away from the zero-recoil point at $q^2 \equiv (p - p')^2 = q_{max}^2$. “ p_μ ” is the momentum of the decaying heavy meson and “ p'_μ ” that of the light daughter meson. High momenta introduce both noise and ap discretization errors into lattice simulations making calculations of semileptonic form factors that extend over a wide range in q^2 a challenging task. We have carried out exploratory simulations to investigate whether anisotropic lattices can help with the signal-to-noise problem. We study two- and three-point correlators involving finite momentum hadrons on both anisotropic and isotropic lattices with comparable coarse spatial lattice spacings. Working with identical source operators and smearings on the initial time slices, we compare the quality of signals that can be extracted at later times. We find that it is considerably easier to obtain good signals on anisotropic lattices at higher momenta (e.g. once momenta reach $(1,1,1)2\pi/(a_s L)$ and beyond).

2. Some Computational Details

The gauge actions on the isotropic and anisotropic lattices correspond to Symanzik im-

proved actions and include both plaquettes $P_{\mu\nu}$ and rectangles $R_{\mu\nu}$ [1].

$$\mathcal{S}_G^{(iso)} = -\beta \sum_{x, \mu > \nu} \left\{ \frac{5}{3} \frac{P_{\mu\nu}}{u_L^4} - \frac{1}{12} \frac{R_{\mu\nu}}{u_L^6} - \frac{1}{12} \frac{R_{\nu\mu}}{u_L^6} \right\} \quad (1)$$

$$\mathcal{S}_G^{(aniso)} = -\beta \sum_{x, s > s'} \frac{1}{\chi_0} \left\{ \frac{5}{3} \frac{P_{ss'}}{u_s^4} - \frac{1}{12} \frac{R_{ss'}}{u_s^6} - \frac{1}{12} \frac{R_{s's}}{u_s^6} \right\} -\beta \sum_{x, s} \chi_0 \left\{ \frac{4}{3} \frac{P_{st}}{u_s^2 u_t^2} - \frac{1}{12} \frac{R_{st}}{u_s^4 u_t^2} \right\}. \quad (2)$$

We use the Landau link definition for the tadpole improvement factors u_L , u_s and u_t . χ_0 is the bare anisotropy. It is related to the renormalized anisotropy $\chi = a_s/a_t$ as, $\chi_0 = \chi/\eta$. η must be fixed using some physics criterion, such as the requirement of a correct relativistic dispersion relation for torelons [2]. For the light quarks we use the D234 action [3],

$$\begin{aligned} \mathcal{S}_{D234} = & \sum_x \bar{\Psi} \left\{ \gamma_t (\nabla_t - \frac{1}{6} C_{3t} \nabla_t^{(3)}) \right. \\ & + \frac{C_0}{\chi} \vec{\gamma} \cdot (\vec{\nabla} - \frac{1}{6} C_3 \vec{\nabla}^{(3)}) + a_t m_0 \\ & - \frac{r}{2} \left[\chi (\nabla_t^{(2)} - \frac{1}{12} C_{4t} \nabla_t^{(4)}) + \frac{1}{\chi} \sum_j (\nabla_j^{(2)} \right. \\ & \left. \left. - \frac{1}{12} C_4 \nabla_j^{(4)}) \right] - r \frac{C_F}{4} i \sigma_{\mu\nu} \tilde{F}^{\mu\nu} \frac{a_s a_t}{a_\mu a_\nu} \right\} \Psi. \quad (3) \end{aligned}$$

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Table 1
Simulation Details.

	isotropic	anisotropic
lattice size	$8^3 \times 20$	$8^3 \times 48$
# configs	200	200
β	1.719	1.8
χ_0	1	6.0
$\chi = a_s/a_t$	1	5.3 [2]
C_0	1	0.82 [4]
a_s^{-1}	0.8(1) GeV [3]	0.7(1) GeV [2]
a_t^{-1}	0.8(1) GeV	3.7(4) GeV
$a_t m_0$	1.15	0.39
P/V	0.725(5)	0.726(6)
$a_s M_0$	6.5	7.0 and 2.0

For the isotropic simulations χ and all the C_j coefficients are set equal to one. On anisotropic lattices we drop discretization corrections in the temporal direction and $C_{3t} = C_{4t} = 0$. The coefficient C_0 must be tuned to ensure correct dispersion relations in correlators involving light propagators. We use a one-loop perturbative estimate for C_0 in our anisotropic simulations [4]. The other coefficients C_3 , C_4 and C_F are set equal to one. For the heavy quarks we employ the NRQCD action [5], suitably modified to allow for an anisotropy.

In Table I we summarize simulation parameters. In this exploratory study we have not attempted to tune quark masses very carefully. We work at one light quark mass slightly heavier than the strange quark. The heavy quark mass is chosen so that the heavy-light pseudoscalar meson mass is close to that of the physical B_s . On the anisotropic lattice we also worked at a second heavy quark mass value close to the charm.

3. Two Point Correlators

In Figure 1. we show examples of extracted finite momentum energies from the pion and rho channels at momentum $(1,1,1)2\pi/(a_s L)$. Since identical signals are being created on the initial time slices, effective mass plateaus are of the

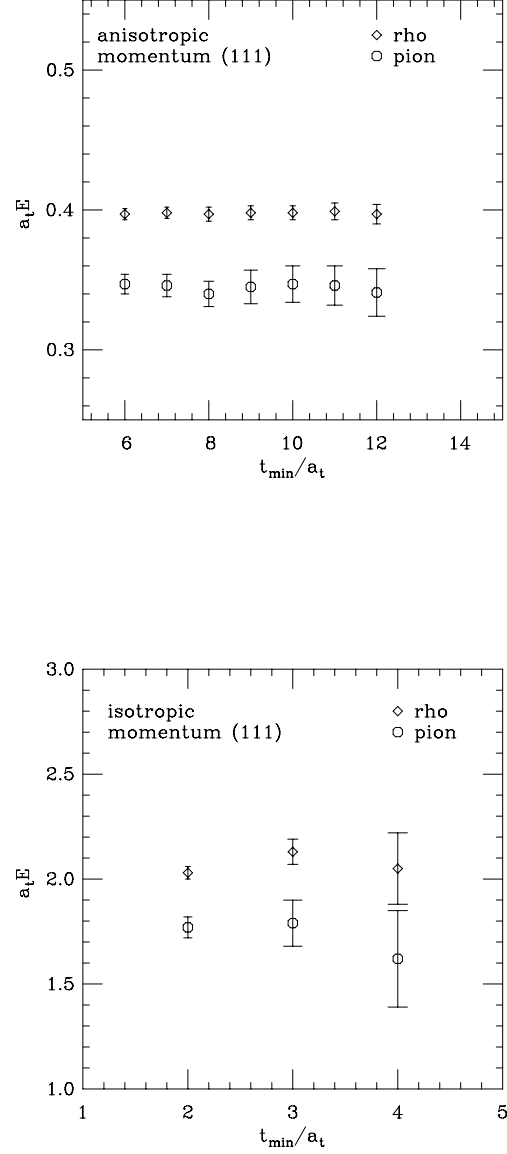


Figure 1. Fitted energies versus t_{\min}/a_t for the pion and the rho for momentum $(1,1,1)$ on anisotropic (top figure) and isotropic (bottom figure) lattices.

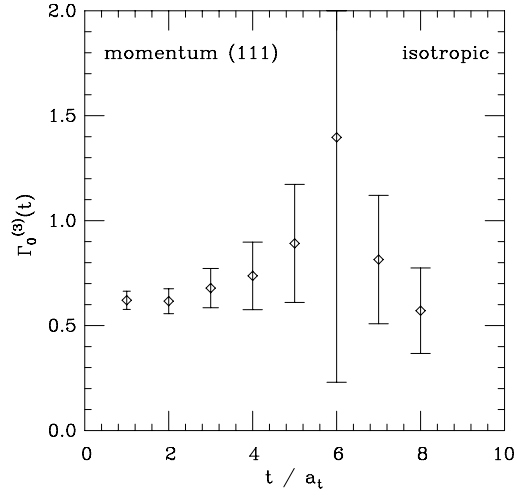


Figure 2. $\Gamma_0^{(3)}(t)$ of eq.(5) for pion momentum (1,1,1) from isotropic lattices versus time in lattice units.

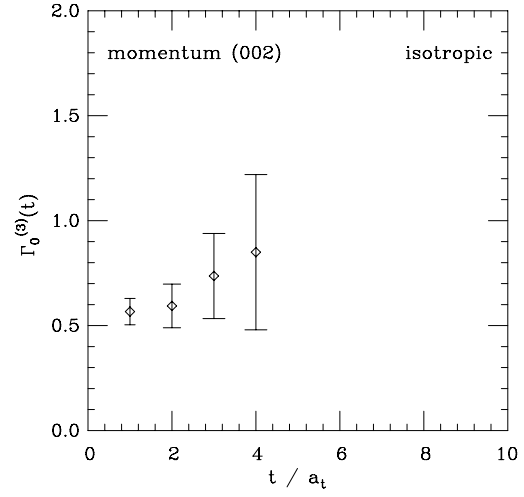


Figure 4. $\Gamma_0^{(3)}(t)$ of eq.(5) for pion momentum (0,0,2) from isotropic lattices versus time in lattice units.

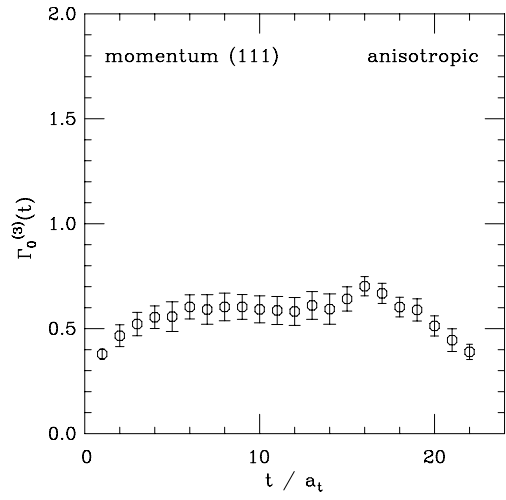


Figure 3. Same as Fig. 2. from anisotropic lattices.

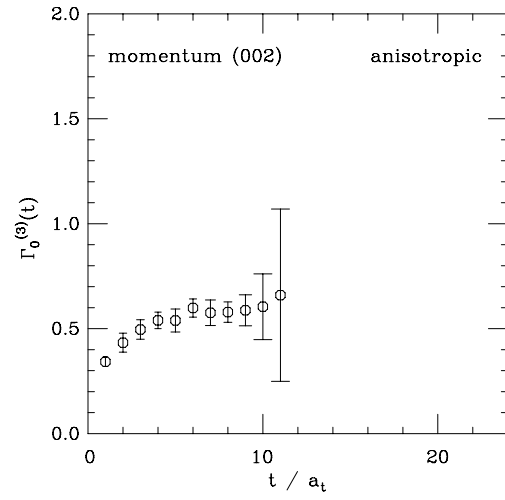


Figure 5. Same as Fig. 4. from anisotropic lattices.

same length in physical units on the isotropic and anisotropic lattices. However, on the anisotropic lattice the region where the signal is still appreciably above the noise is being probed much more frequently. It is then easier to recognize the onset of a plateau and extract fitted energies with confidence.

4. Three Point Correlators

Semi-leptonic form factors for pseudoscalar to pseudoscalar decays are defined through the hadronic matrix element of the heavy-light vector current V_μ ,

$$\langle \pi(p') | V^\mu | B(p) \rangle = f_+(q^2)(p^\mu + p'^\mu) + f_-(q^2)(p^\mu - p'^\mu). \quad (4)$$

In lattice simulations the above matrix element can be obtained from ratios of three- and two-point correlators. We define,

$$\Gamma_\mu^{(3)}(\vec{p}, \vec{p}', t_B, t) \equiv \frac{G_\mu^{(3)}(\vec{p}, \vec{p}', t_B, t)}{G_B^{(2)}(\vec{p}, t_B - t) G_\pi^{(2)}(\vec{p}', t)} \sqrt{\zeta_{BB} \zeta_{\pi\pi}} \quad (5)$$

$$\rightarrow \frac{\langle B(\vec{p}) | V^\mu | \pi(\vec{p}') \rangle}{2\sqrt{E_B E_\pi}}, \quad (6)$$

where,

$$G_\mu^{(3)}(\vec{p}, \vec{p}', t_B, t) = \sum_{\vec{x}} \sum_{\vec{y}} e^{-i\vec{p} \cdot \vec{x}} e^{i(\vec{p} - \vec{p}') \cdot \vec{y}} \langle 0 | \Phi_B(t_B, \vec{x}) V_\mu^L(t, \vec{y}) \Phi_\pi^\dagger(0) | 0 \rangle \quad (7)$$

and

$$G_B^{(2)}(\vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \langle 0 | \Phi_B(t, \vec{x}) \Phi_B^\dagger(0) | 0 \rangle \rightarrow \zeta_{BB} e^{-E_B^{sim} t}, \quad (8)$$

$$G_\pi^{(2)}(\vec{p}', t) \rightarrow \zeta_{\pi\pi} e^{-E_\pi t}. \quad (9)$$

Φ_B and Φ_π are interpolating operators for the B and π mesons. V_μ^L is the lattice heavy-light vector current appropriately matched to the continuum. Figures 2. - 5. show $\Gamma_{\mu=0}^{(3)}$ versus t/a_t at fixed t_B for two different pion momenta \vec{p}' on the two lattices. The B meson momentum is kept at

$\vec{p} = 0$. One sees again that the anisotropy greatly facilitates extracting a reliable signal. From this data one can use eq.(4) and eq.(6) to determine the form factors $f_\pm(q^2)$. Results for form factors will be presented elsewhere [6].

5. Summary

We find evidence that anisotropic simulations allow us to go to higher momenta in two- and three-point hadronic correlators. For studies of semi-leptonic heavy meson decays this means that a wider range in $q^2 = (p - p')^2$ can be covered. As part of the present project we have also tested for ap discretization errors in our simulations. We have looked at dispersion relations and the effects of nonzero B meson momenta on the decay constant f_B . We find good continuum behavior up to $a_s p \approx 1.5$ and only 5-10% deviations up to about $a_s p \sim 2$. These tests will also be summarized in a forthcoming paper together with the form factor results [6]. There we will go into details of weighing the additional costs associated with anisotropic simulations against the advantages described here.

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